Durable goods in price index measurement

Revisiting the user cost approach

Craig Pesme

Paris School of Economics - ‘Measurement in Economics’ Chair

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Motivation

- Climate transition: necessity to replace brown goods by green substitutes
- Often durable goods: cars, boilers...
- How much will it cost households ?

$\rightarrow$ Role of real income and price indices to measure this

Already a big literature on durable goods

- See for instance Diewert et al. (2020)
- But the approach always seems a bit ‘manual’
  - ‘We *imagine* that the consumer purchases the good during period t and then sells it during the following period (possibly to himself)’
- Sometimes confusion with the approach, many different variations of the formula, and criticisms
  - nominal or real interest rate ? start or end of period user cost ?

This presentation

1. Can we formalize the user cost approach by extending the usual cost-of-living-index theory ?
2. What are the implications for
   - the treatment of second-hand markets ?
   - the impact of a cost increase of durables on inequality ?
3. Some illustrations with the automobile market in France

A simple one-period framework

Assume :

1. Consumer have preferences over market commodities x, represented by utility function $U : x \to U(x)$
2. At each period, they face prices p_t, receive income I_t, and solve:

$$V(p_t, I_t) = \max_x U(x)$$
$$\text{s.t. } p_t x = I_t$$

The usual Konüs cost-of-living-index (COLI)

- The Konüs cost-of-living-index between two periods, for a reference utility level u, is given by :

$$\mathcal{P}^{K}_{1/0} = \frac{e(p_1, u)}{e(p_0, u)}$$

where e(.) is the expenditure function:

$$e(p, u) = \min I \text{ s.t. } V(p, I) \geq u$$

- From there, real income growth can be defined as :

$$\mathcal{G}_{1/0} = \frac{I_1}{I_0} / \mathcal{P}^{K}_{1/0}$$

Taking it the other way around

- First define **real income** as equivalent income (or money-metric utility), for a reference price vector $\tilde{p}$

$$\tilde{I}\big((p, I)|\tilde{p}\big) = e\big(\tilde{p}, V(p, I)\big)$$

- Growth of real income:

$$\mathcal{G}_{1/0} = \frac{\tilde{I}\big((p_1, I_1)|\tilde{p}\big)}{\tilde{I}\big((p_0, I_0)|\tilde{p}\big)}$$

- **Cost-of-living-index**:

$$\mathcal{P}_{1/0} = \frac{I_1}{I_0} \Big/ \frac{\tilde{I}\big((p_1, I_1)|\tilde{p}\big)}{\tilde{I}\big((p_0, I_0)|\tilde{p}\big)}$$

$\rightarrow$ By definition $\mathcal{G}_{1/0} = \frac{I_1}{I_0} / \mathcal{P}_{1/0}$

The usual result under the assumption of homotheticity

Let the Divisia price index (continuous version of a chain-linked index) be defined as:

$$\mathcal{P}^D_{1/0} = \exp \int_0^1 \frac{p'(s)x(s)}{p(s)x(s)} ds$$

If preferences are homothetic, then:

$$\mathcal{P}^K = \mathcal{P} = \mathcal{P}^D$$

Adding capital and savings

- In reality, consumers have intertemporal preferences on top of their 'current consumption' preferences
- Each period, they trade off consumption and savings by solving an intertemporal program:

$$Z\left(p_t, r_t, I_t, K_t\right) = \max_{x,S} W\left(U(x), (1+r_t)(K_t + S)\right)$$
$$\text{s.t. } p_t x + S = I_t$$

  where K_t is capital inherited from the previous period, and r_t is the observed interest rate.
- The function $W(.)$ also implicitly depends on anticipations of future conditions (prices, future income...)

Adding durables

With durable goods, the problem is thay they appear in both arguments of the function $W()$:

$$Z\left(p_t, r_t, I_t, K_t\right) = \max_{x, u_d, S} W\Big(U(x, u_d); (1+r_t)(K_t + S) + I_t^d \mathbb{1}_{u_d>0}(1-\delta_t)\Big)$$

$$\text{s.t. } p_t x + I_t^d \mathbb{1}_{u_d>0} + S = I_t$$

But this can be re-written:

$$Z\left(p_t, r_t, I_t, K_t\right) = \max_{x, u_d, \tilde{s}} W\Big(U(x, u_d); (1+r_t)(K_t + \tilde{S})\Big)$$

$$\text{s.t. } p_t x + \frac{r_t + \delta_t}{r_t + 1} I_t^d \mathbb{1}_{u_d>0} + \tilde{S} = I_t$$

By making the 'durable good' problem disappear, the user cost formula popped up.

How to deal with savings ? (1/2)

Dealing with durable goods boils down to how we want to treat savings in the definition of real income. Two possibilities:

1. Ignore savings and measure the maximum amount of consumption that income can buy, ie. follow

$$V(p_t, I_t) = \max_x U(x) \text{ s.t. } p_t x = I_t$$

- this is what our current measures are doing
- need homotheticity assumption to be correct
- does not take into account the fact that the 'purchasing power' of income also comes from its capacity to buy future consumption, through savings.

How to deal with savings ? (2/2)

2. Take savings into account in the definition of real income
   - Write the program as:

$$Z\left(p_t, r_t, I_t, K_t\right) = \max_{x,S} W\left(U(x), (1 + r_t) \times \frac{S + K_t}{\mathcal{P}_{t/0}}\right)$$

$$\text{s.t. } p_t x + S = I_t$$

   where $\mathcal{P}_{t/0}$ is the price index for commodities x.
   - $(1 + r_t) \times \frac{S+K_t}{\mathcal{P}_{t/0}}$ corresponds to 'real value of savings and capital'.
   - Define real income $\tilde{I}$ as:

$$e\left(\tilde{p}, \tilde{r}, K_t; Z\left(p_t, r_t, I_t, K_t\right)\right)$$

In both cases, the price index is the usual one

- With definition 1:

$$\frac{\tilde{I}_1}{\tilde{I}_0} = \frac{I_1}{I_0} / \mathcal{P}^D_{1/0}$$

- With definition 2:

$$\frac{\tilde{I}_1}{\tilde{I}_0} \approx \left(\frac{I_1}{I_0} / \mathcal{P}^d_{1/0} \right) \times \left(\frac{1 + r_1}{1 + r_0} \right)^{\tau^0_S}$$

  where τ^0_S is the savings rate in 0.
- In both cases, the price index is the usual Divisia index.

Conclusion: price index with durable goods

Recall that:

$$Z(p_t, r_t, I_t) = \max_{x, u_d, \tilde{s}} W\Big(U(x, u_d); (1+r_t)K_t + (1+r_t)\tilde{S}\Big)$$

$$\text{s.t. } p_t x + \frac{r_t + \delta_t}{r_t + 1} I_t^d \mathbb{1}_{u_d > 0} + \tilde{S} = I_t$$

The natural extension from non-durables to durables is to use a user cost approach.

Implication for second-hand market

The second-hand market is usually ignored in the price index:

- Compensation between households argument holds with the acquisitions approach
- But not with the user cost approach. Take a geometric depreciation model and assume a stationary state so that purchases X_d are stable over time. Then :

$$\Big(\sum_{t=0}^{\infty} \frac{r+\delta}{r+1}(1-\delta)^t I^d\Big) X_d = \frac{r+\delta}{r+1}\frac{1}{\delta} I^d$$

- The total expenditure on vehicles depends on the depreciation rate δ
- Lower $\delta \rightarrow$ bigger expenditure level (intuition: very high $\delta \rightarrow$ the good is almost bought as a non-durable)

Implications for costs borne by households (1/2)

What would be the consequence of a price increase of a durable good ?

- From an aggregate point of view, average cost is $\frac{r+\delta}{r+1}\frac{1}{\delta}I^d$
  1. negative effect through direct channel
  2. negative effect through indirect channel of δ

Intuition for the indirect effect: assume preferences between older and newer cars are of the form

$$U\Big(x, u_d(a) + \alpha u_d(a+1)\Big)$$

where $\alpha < 1$
Then δ is an increasing function of the ratio $\frac{c}{I}$, where I is the acquisition cost and c are costs of use.

Implications (1/2): example with automobile market

Based on France Stratégie (2022) assumptions (except $r = 6\%$)

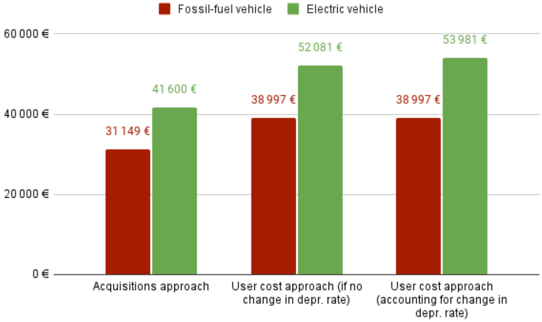

Figure: Acquisitions vs. User cost approach

The change in δ (from 18% to 15%) was estimated using the form $U\left(u_d(a) + \alpha u_d(a+1)\right)$ and the current observed depreciation rate.

Implications for costs borne by households (2/2)

From a disaggregated perspective, the price increase of a durable good:

- decreases the depreciation rate δ
- which is good for households who buy newer cars
- bad for the others

Implications (2/2): example with automobile market

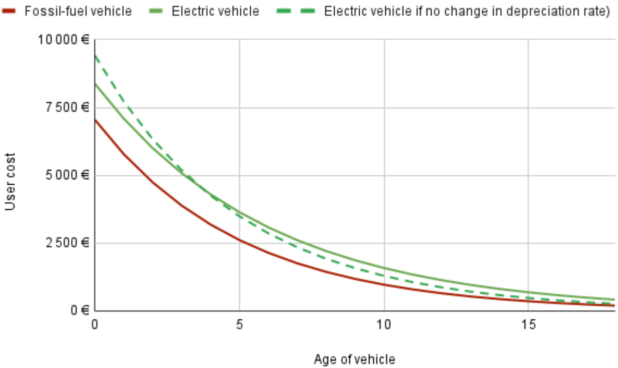

Figure: User cost by vehicle age

Richer households own more recent cars

Figure taken from Service des données et études statistiques (SDES):

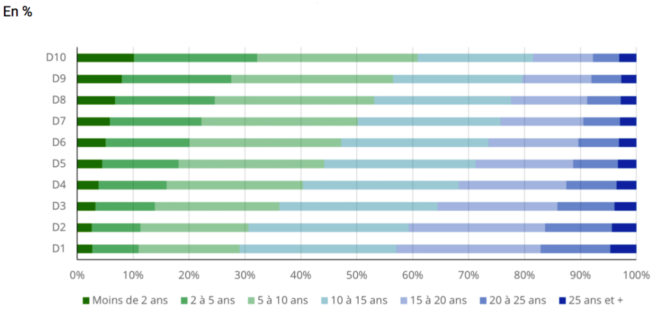

Figure: Age of owned cars by household living standards (2023)

Dis-aggregated impact of an acquisition cost increase

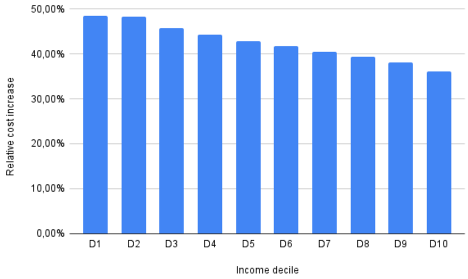

Figure: Cost increase by household living standards

- If no change in depreciation rate, all columns would be the same
- This inflation inequality comes on top of inequality due to non-homotheticity.

Thank you for your attention